SEMESTER-II

PERIOD-IV

TOPIC

6

Linear Equations and Inequalities

6.1 EQUALITY *AND* EQUIVALENCE

Formation of Linear Equations

In Semester-I, you have learnt the formation of algebraic expression under given conditions. Let us expand our study to form linear equations by using concept of equality.

Example 1. *Make an equation for each of the given conditions:*

(*i*) *Sum of a number x and 5 is 25.*

(*ii*) *Take away 3 from a number y gives twice the number.*

(*iii*) *x multiplied by 5 gives 1.*

(*iv*) *The product of –2 and a number p is equal to 0.*

(*v*) *The quotient of y divided by 4 is –10.*

Solution.

(*i*) Sum of a number x and 5 is $x + 5$. But this sum is equal to 25.

$\therefore \quad x + 5 = 25$

(*ii*) Take away 3 from a number y is $y - 3$.

Twice the number y is $2y$.

These two results are equal.

$\therefore \quad y - 3 = 2y$

(*iii*) x multiplied by 5 is $5 \times x$ or $5x$.

This product gives 1.

$\therefore \quad 5x = 1.$

(*iv*) The product of –2 and p is $-2p$.

This is equal to 0.

$\therefore \quad -2p = 0$

(*v*) The quotient of y divided by 4 is $\frac{y}{4}$.

This equals to –10.

$\therefore \quad \frac{y}{4} = -10.$

Other examples of linear equations in one variable are:

Linear Equations in one variable	Variable
$3x - 2 = 4$	x
$x + 12 = 2x - 3$	x
$16y = y + 15$	y
$y - 4 = 4$	y
$-8z + 2 = -10$	z

A linear equation is an equation which contains the variable which is not raised to any power other than 1.

Every equation consits of left hand side (LHS), an equality sign (=) and right hand side (RHS). Look at this example:

Equality sign

$5x - 6 = 19$

LHS RHS

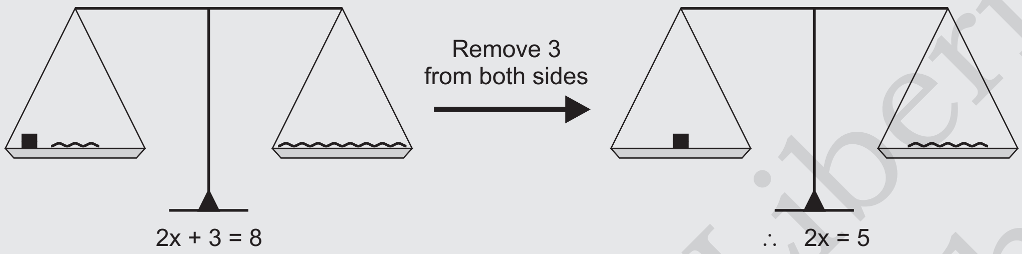
Here, LHS is $5x - 6$ and RHS is 19. In every equation LHS and RHS are always equal.

Properties of Equalities and Equivalence

Two equations that have the same solution are called *equivalent* equations *e.g.* 4 + 3 = 2 + 5. And this as we learned in the above sub-section *i.e.*, formation of linear equations is shown by the equality sign =. An inverse operation are two operations that undo each other *e.g.* addition and subtraction or multiplication and division. You can perform the same inverse operation on each side of an equivalent equation without changing the equality.

ACTIVITY 1

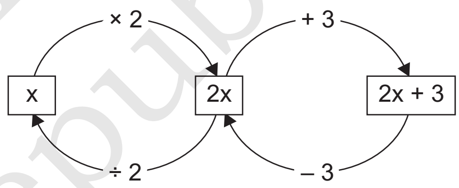
Compare the balance of weights:



We perform operations on both sides of each equation in order to *isolate the unknown.*

We consider how the expression has been *built up* and then *isolate the unknown* by using *inverse operations* in *reverse order.*

Consider the equation $2x + 3 = 8$, the LHS is built up by starting with x, multiplying by 2, then adding 3.



So, to isolate x, we first subtract 3 from both sides, then divide both sides by 2.

Example 2. *Solve for x:* $3x + 7 = 22$.

Solution. Here, $3x + 7 = 22$

$\therefore \quad 3x + 7 - 7 = 22 - 7$ (Subtracting 7 from both sides)

$\therefore \quad 3x = 15$ (Simplifying)

$\therefore \quad \frac{3x}{3} = \frac{15}{3}$ (Dividing both sides by 3)

$\therefore \quad x = 5$ (Simplifying)

Check: LHS $= 3 \times 5 + 7 = 22$ $\therefore$ LHS = RHS.

EXERCISE 6.1

1. Solve for x:

(*a*) $x + 11 = 0$ (*b*) $4x = -12$ (*c*) $5x + 35 = 0$ (*d*) $4x - 5 = -17$

2. Solve for x:

(*a*) $8 - x = -3$ (*b*) $-4x = 22$ (*c*) $3 - 2x = 11$ (*d*) $6 - 4x = -8$

3. Solve for x:

(*a*) $\frac{x}{4} = 7$ (*b*) $\frac{2x}{5} = -6$ (*c*) $\frac{x}{2} + 3 = -5$ (*d*) $\frac{x}{4} - 2 = -5$

4. Solve for x:

(*a*) $\frac{2x+11}{3} = 0$ (*b*) $\frac{1}{2}(3x+1) = -4$

(*c*) $\frac{1+2x}{5} = 7$ (*d*) $\frac{1-2x}{5} = 3$.

6.2 FINDING THE SOLUTION SET OF A LINEAR EQUATION

Solving an equation is the process of finding the solutions of the equation. The set of all solutions of an equation is called the *solution set* of the equation.

For example: $x = 3$ is a solution of the equation $4x - 9 = 3$ because, substituting $x = 3$ in the equation, we have $4 \times 3 - 9 = 3$ or $12 - 9 = 3$ or $3 = 3$ which is true.

Therefore, the solution set of the equation $4x - 9 = 3$ is $S = \{x : x = 3\}$ or $\{3\}$.

The solution set of a linear equation is a *singleton set.*

Equations having the same solution set are called *equivalent equations.*

For example: $4x - 3 = 9$, $4x - 9 = 3$, $4x = 12$ are equivalent equations.

Rules for solving linear equations in one variable

To solve a linear equation in one variable, we transform the given equation into an equivalent equation of the form $x = k$, where $k \in R$. Then the solution set is $S = \{x : x = k\}$ or $\{k\}$.

In an equation, the two sides are equal, *i.e.*, balanced. Performing the same mathematical operations on *both sides* of the equation does not disturb the balance and an equivalent equation is generated.

Example 3. *Solve 6x – 7 = 23 and check your solution.*

Solution. Given: $6x - 7 = 23$

Adding 7 to both sides	*OR* Transposing –7 to RHS
$6x - 7 + 7 = 23 + 7$	$6x = 23 + 7$
$\Rightarrow \quad 6x = 30$	$\Rightarrow \quad 6x = 30$

Dividing both sides by 6, we get

$$\frac{6x}{6} = \frac{30}{6} \quad \Rightarrow \quad x = 5 \quad \text{is the required solution.}$$

The solution set is S = $\{x : x = 5\}$ or $\{5\}$.

Check: Replacing x by 5 in the given equation, we have

$$6 \times 5 - 7 = 23$$

or $$30 - 7 = 23, \quad \text{which is true.}$$

Example 4. *Solve:* $\frac{3x}{5} - \frac{x}{3} = \frac{x}{6} + 1\frac{1}{2}.$

Solution. Given: $\frac{3x}{5} - \frac{x}{3} = \frac{x}{6} + \frac{3}{2}$ $\left[\because \; 1\frac{1}{2} = \frac{3}{2}\right]$

Multiplying by 30, the LCM of 5, 3, 6 and 2, we get

$$6(3x) - 10x = 5x + 15(3)$$

$$\Rightarrow \quad 18x - 10x = 5x + 45 \quad \Rightarrow \quad 8x = 5x + 45$$

$$\Rightarrow \quad 8x - 5x = 45 \quad \Rightarrow \quad 3x = 45 \quad \Rightarrow \quad x = \frac{45}{3} = 15.$$

The solution set is S = $\{15\}$.

Example 5. *Solve:* $\frac{5x-2}{3} = \frac{3x+2}{2}.$

Solution. Given: $\frac{5x-2}{3} = \frac{3x+2}{2}.$

By cross-multiplication (or multiplying both sides by 6, the LCM of 3 and 2), we have

$$2(5x - 2) = 3(3x + 2)$$

$$\Rightarrow \quad 10x - 4 = 9x + 6$$

$$\Rightarrow \quad 10x - 9x = 6 + 4$$

$$\Rightarrow \quad x = 10, \text{ is the required solution.}$$

$\therefore$ The solution set is S = $\{10\}$.

EXERCISE 6.2

1. Verify that $x = 6$ is a solution of the equation

$$2(x - 3) - 17 = 13 - 3(x + 2).$$

2. Verify that $x = 8$ is a solution of the equation $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}.$

3. Solve the following equations and check your solution:

(*a*) $4x - 7 = 9$ (*b*) $3x + 11 = 2$

(*c*) $7 - 2(5 - 3x) = 4(x - 3) + 5$ (*d*) $7(x - 2) = 2(2x - 4)$

4. Solve the following equations and check your solution:

(*a*) $\frac{5x}{3} + \frac{2}{5} = 1$ (*b*) $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 13$

(*c*) $\frac{2x - 1}{3} - \frac{6x - 2}{5} = \frac{1}{3}$ (*d*) $\frac{2x - 3}{6} - \frac{x - 5}{2} = \frac{x}{6}$

6.3 WORD PROBLEMS INVOLVING LINEAR EQUATIONS

Problems stated in words are called *word problems.* Solving word problems involves two steps:

(*i*) formulation (*ii*) solution

The following steps should be followed to solve a word problem.

(*a*) Read the problem carefully and note what is given and what is required.

(*b*) Denote the unknown quantity by x.

(*c*) Translate the statement of the problem into mathematical statements.

(*d*) Use the given conditions to form an equation.

(*e*) Solve the equation.

Example 6. *A number is such that it is as much greater than 84 as it is smaller than 108. Find the number.*

Solution. Let the number be x. Then, the number is greater than 84 by $x - 84$. Also, the number is smaller than 108 by $108 - x$.

Given $x - 84 = 108 - x$

$\Rightarrow$ $x + x = 108 + 84$

$\Rightarrow$ $2x = 192$

$\Rightarrow$ $x = \frac{192}{2} = 96$

Hence the required number is 96.

Example 7. *The present age of Ella's mother is three times the present age of Ella. After 5 years their ages will add to 66 years. Find their present ages.*

Solution. Let Ella's present age be x years, then the present age of Ella's mother is $3x$ years.

After 5 years, Ella's age will be $(x + 5)$ years and her mother's age will be $(3x + 5)$ years.

By the given condition

$$(x + 5) + (3x + 5) = 66 \quad \Rightarrow \quad x + 5 + 3x + 5 = 66$$

$$\Rightarrow \quad 4x + 10 = 66 \quad \Rightarrow \quad 4x = 66 - 10$$

$$\Rightarrow \quad 4x = 56 \quad \Rightarrow \quad x = \frac{56}{4} = 14$$

Therefore, the present age of Ella = 14 years and the present age of Ella's mother = 3 × 14 = 42 years.

Example 8. *The denominator of a fraction exceeds its numerator by 4. If the numerator and the denominator both are increased by 3, the fraction becomes* $\frac{4}{5}$. *Find the original fraction.*

Solution. Let the numerator of the original fraction be x, then its denominator is $(x + 4)$.

The original fraction = $\frac{x}{x+4}$

When the numerator and denominator both are increased by 3, the new fraction = $\frac{x+3}{(x+4)+3} = \frac{x+3}{x+7}$

By the given condition

$$\frac{x+3}{x+7} = \frac{4}{5}$$

By cross-multiplication

$$5(x + 3) = 4(x + 7) \quad \Rightarrow \quad 5x + 15 = 4x + 28$$

$$\Rightarrow \quad 5x - 4x = 28 - 15 \quad \Rightarrow \quad x = 13$$

Therefore, the original fraction = $\frac{13}{13+4} = \frac{13}{17}$.

EXERCISE 6.3

1. If $\frac{1}{3}$ of a number is added to $\frac{1}{5}$ of the same number, the result is 8. Find the number.
2. The sum of three consecutive even numbers is 24. Find the numbers.

3. When a certain number is subtracted from 10 and the result is multiplied by 2, the final result is 4. Find the number.
4. The sum of five consecutive odd numbers is 35. Find the numbers.
5. $\frac{5}{6}$ of the number of pupils in a class is 4 greater than three-quarters of the number in the class. Find the number of pupils in the class.
6. A certain car covers 10 km at a certain speed. If this average speed is reduced by 30 km/h, the car takes the same time to cover a distance of 6 km. Find the speed of the car in the first part of the journey.

6.4 LINEAR INEQUALITIES IN ONE VARIABLE

Two algebraic expressions related by the symbol < (is less than) or > (is greater than) or ≤ (is less than or equal to) or ≥ (is greater than or equal to) form an *inequality.*

An inequality in any one of the following forms:

$$\left.\begin{array}{ll} ax + b < 0 & \quad ...(1) \\ ax + b > 0 & \quad ...(2) \\ ax + b \le 0 & \quad ...(3) \\ ax + b \ge 0 & \quad ...(4) \end{array}\right\} \quad \begin{array}{l} a, b \in R \\ a \ne 0 \end{array}$$

is called a *linear inequality in one variable x.*

(1) and (2) are called *strict inequalities* while (3) and (4) are called *slack inequalities.*

Thus, $2x + 3 < 0$, $3x - 4 > 0$, $7x - 2 \le 0$
$5x + 2 \ge 0$ are linear inequalities in x.

6.5 SOLVING LINEAR INEQUALITIES IN ONE VARIABLE

To solve a linear inequality in one variable, we transform the given inequality into an equivalent inequality of the form $x < k$ or $x > k$ or $x \le k$ or $x \ge k$ by the following rules:

(*i*) If the same quantity is added to or subtracted from both sides of an inequality, then the sign of the inequality is not affected.

For example:

$$a < b \;\Rightarrow\; a + c < b + c$$
$$a \ge b \;\Rightarrow\; a - c \ge b - c$$

(*ii*) If both sides of an inequality are multiplied or divided by a *positive number,* then the sign of inequality is not affected.

For example:

$$a \geq b \quad \text{and} \quad c > 0 \quad \Rightarrow \quad ac \geq bc$$

$$a < b \quad \text{and} \quad c > 0 \quad \Rightarrow \quad \frac{a}{c} < \frac{b}{c}.$$

(*iii*) If both sides of an inequality are multiplied or divided by a *negative number,* then the direction of inequality is reversed, *i.e.*, < changes into > and vice versa.

For example:

$$a < b \quad \text{and} \quad c < 0 \quad \Rightarrow \quad ac > bc$$

$$a \geq b \quad \text{and} \quad c < 0 \quad \Rightarrow \quad \frac{a}{c} \leq \frac{b}{c}.$$

Thus, always reverse the sign of inequality when multiplying or dividing by a negative number.

(*iv*) A term may be transposed from one side of the inequality to the other side by changing its sign.

Example 9. *Solve for x:* $-5 < 9 - 2x$.

Solution. $-5 < 9 - 2x$

$\therefore$ $-5 + 2x < 9 - 2x + 2x$ [Adding $2x$ to both sides]

$\therefore$ $2x - 5 < 9$

$\therefore$ $2x - 5 + 5 < 9 + 5$ [Adding 5 to both sides]

$\therefore$ $2x < 14$

$\therefore$ $\dfrac{2x}{2} < \dfrac{14}{2}$ [Dividing both sides by 2]

$\therefore$ $x < 7$

Check: If $x = 5$ then $-5 < 9 - 2 \times 5$, *i.e.,* $-5 < -1$ which is true.

Example 10. *Solve for x:* $3 - 5x \geq 2x + 7$.

Solution. $3 - 5x \geq 2x + 7$

$\therefore$ $3 - 5x - 2x \geq 2x + 7 - 2x$ [Subtracting $2x$ from both sides]

$\therefore$ $3 - 7x \geq 7$

$\therefore$ $3 - 7x - 3 \geq 7 - 3$ [Subtracting 3 from both sides]

$\therefore$ $-7x \geq 4$

$\therefore \qquad \dfrac{-7x}{-7} \le \dfrac{4}{-7}$

[Dividing both sides by –7, so reverse the sign]

$\therefore \qquad x \le \dfrac{4}{7}$

Check: If $x = -1$ then $3 - 5 \times (-1) \ge 2 \times (-1) + 7$, *i.e.*, $8 \ge 5$ which is true.

6.6 GRAPH OF LINEAR INEQUALITIES IN ONE GRAPH

Solutions of inequalities can be represented on a number line as follows:

(*i*) Graph of $x < 5$, $x \in$ N

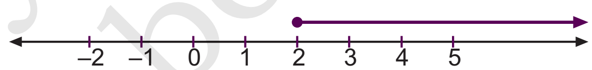
The solution set = {1, 2, 3, 4} is shown by thick dots on the number line. It consists of four isolated points.

(*ii*) Graph of $-3 \le x < 4$, $x \in$ I

The solution set = {–3, –2, –1, 0, 1, 2, 3} is shown by thick dots on the number line.

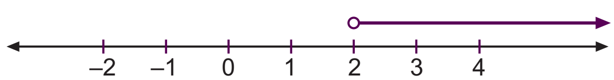
(*iii*) Graph of $x \ge 2$, $x \in$ R

The solution set = $\{x : x \ge 2, x \in R\}$ is shown by a rightward arrow emanating from 2. The shaded circle above 2 indicates '2 is included'.



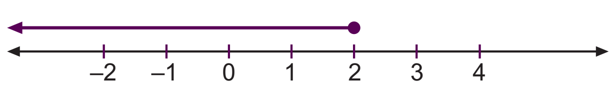
(*iv*) Graph of $x > 2$, $x \in$ R

The solution set = $\{x : x > 2, x \in R\}$ is shown by a rightward arrow emanating from 2. The unshaded circle above 2 indicates '2 is not included'.



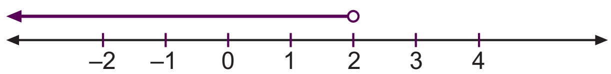
(*v*) Graph of $x \le 2$, $x \in$ R

The solution set = $\{x : x \le 2, x \in R\}$ is shown by a leftward arrow emanating from 2. The shaded circle above 2 indicates '2 is included'.



(*vi*) Graph of $x < 2$, $x \in$ R

The solution set = $\{x : x < 2, x \in R\}$ is shown by a leftward arrow emanating from 2. The unshaded circle above 2 indicates '2 is not included'.



(*vii*) Graph of $-2 < x \leq 3$, $x \in$ R

The solution set = $\{x : -2 < x \leq 3, x \in R\}$ is shown by a line segment whose left end point above –2 is not included (shown by an unshaded circle) and right end point above 3 is included (shown by a shaded circle).

Example 11. *Solve: $3x + 1 \leq 16$, $x \in$ **N** and represent the solution on the number line.*

Solution. Given $3x + 1 \leq 16$

Transposing 1 to RHS

$$3x \leq 16 - 1 \quad \Rightarrow \quad 3x \leq 15$$

Dividing both sides by 3,

$$x \leq 5$$

Since, $x \in$ **N**, therefore, $x = 1, 2, 3, 4, 5$

The solution set = $\{1, 2, 3, 4, 5\}$ is shown by thick dots on the number line.

Example 12. *Solve: $3(x - 1) > 2(x + 2) - 9$ and represent the solution on the number line.*

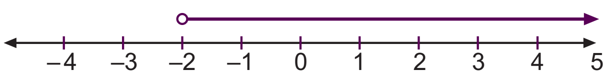
Solution. Given $3(x - 1) > 2(x + 2) - 9$

$\Rightarrow$ $3x - 3 > 2x + 4 - 9 \quad \Rightarrow \quad 3x - 3 > 2x - 5$

Collecting the terms of x on LHS and the constant terms on RHS

$$3x - 2x > -5 + 3 \quad \Rightarrow \quad x > -2$$

The solution set = $\{x : x > -2, x \in R\}$ is shown by a rightward arrow emanating from –2. The unshaded circle above –2 indicates '–2 is not included'.



Example 13. *Solve:* $\frac{2x-1}{3} \le \frac{3x-2}{4} - \frac{2-x}{5}$.

Represent the solution on the number line.

Solution. Given $\frac{2x-1}{3} \le \frac{3x-2}{4} - \frac{2-x}{5}$.

Multiplying both sides by 60, the LCM of 3, 4, 5

$$20(2x-1) \le 15(3x-2) - 12(2-x)$$

$$\Rightarrow \quad 40x - 20 \le 45x - 30 - 24 + 12x$$

$$\Rightarrow \quad 40x - 20 \le 57x - 54$$

Collecting the terms of x on LHS and the constants on RHS

$$40x - 57x \le -54 + 20$$

$$\Rightarrow \quad -17x \le -34$$

Dividing both sides by –17, which is negative

$$x \ge 2$$

The solution set = $\{x : x \ge 2, x \in R\}$ is shown by a rightward arrow emanating from 2. The shaded circle above 2 indicates '2 is included'.

Example 14. *Solve: $0 \le 3x - 1 \le 2$. Represent the solution on the number line.*

Solution. Given $0 \le 3x - 1 \le 2$

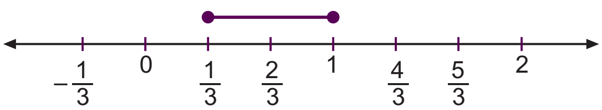
This is a double inequality.

Adding 1 throughout, $1 \le 3x \le 3$

Dividing by 3 throughout, $\frac{1}{3} \le x \le 1$

The solution set = $\left\{x : \frac{1}{3} \le x \le 1, x \in R\right\}$ is shown by a line segment.

The circles above $\frac{1}{3}$ and 1 are both shaded because $\frac{1}{3}$ and 1 are both included in the solution set.



EXERCISE 6.4

1. Solve the following inequalities and represent the solution on the number line:

(*a*) $5x - 3 < 7, x \in \mathbf{N}$ (*b*) $3 - 2x \geq x - 10, x \in \mathbf{N}$

(*c*) $11 + 2x > 5, x \in \mathbf{I}$ (*d*) $4x + 3 < 6x + 7, x \in \mathbf{I}$

2. Solve the following inequalities and represent the solution on the number line:

(*a*) $\frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}$ (*b*) $\frac{x}{4} - \frac{3}{5} \leq \frac{x}{2} + 1$

(*c*) $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$ (*d*) $1 \leq 3(x - 2) + 4 < 10, x \in \mathbf{N}$

3. Illustrate the solution set of the linear inequality $6 - 3x > x + 12$ when x is an integer.

4. Illustrate the solution set of the linear inequality $x + 1 < 4$ on the number line.

5. Find the solution set for the linear inequality $3x + 8 > 2$ and illustrate it on the number line when

(*a*) x is an integer (*b*) x is a real number

6. Find the solution set for linear inequality $7x + 3 > 5x + 9$ and illustrate your answer on the number line, where x is a real number.

7. Illustrate the solution set of the linear inequality $-4 < x \leq 5$ on the "number line.

6.7 WORD PROBLEMS INVOLVING LINEAR INEQUALITIES

Solving word problems involving linear inequalities in one variable involves two steps:

(*i*) formulation (*ii*) solution

Read the problem carefully. Denote the unknown quantity by x. Translate the statement of the problem into mathematical statements. Use the given conditions to form an inequality and then solve it.

Example 15. *Find all pairs of consecutive odd natural numbers which are greater than 12 and their sum is less than 50.*

Solution. Let x and $x + 2$ be two consecutive odd natural numbers.

Given: $x > 12$ and $x + (x + 2) < 50$

$\Rightarrow$ $x > 12$ and $x + x + 2 < 50$

$\Rightarrow \quad x > 12 \quad \text{and} \quad 2x < 50 - 2$

$\Rightarrow \quad x > 12 \quad \text{and} \quad 2x < 48$

$\Rightarrow \quad x > 12 \quad \text{and} \quad x < 24$

$\Rightarrow \quad 12 < x < 24$

Since x is an odd natural number, therefore,

$$x = 13, 15, 17, 19, 21, 23$$

Hence, the required possible pairs are $(x, x + 2)$

$$= (13, 15), (15, 17), (17, 19), (19, 21), (21, 23), (23, 25).$$

Example 16. *On a day, temperature in a city changes from 30° to 35° Celsius (C) in one hour. Find the range of change of temperature in degree Fahrenheit (F) if conversion formula is given by* $C = \frac{5}{9}(F - 32)$.

Solution. Given $30 \le C \le 35$

Putting $C = \frac{5}{9}(F - 32)$, we have

$$30 \le \frac{5}{9}(F - 32) \le 35$$

Multiplying throughout by 9,

$$270 \le 5(F - 32) \le 315$$

Dividing throughout by 5,

$$54 \le F - 32 \le 63$$

Adding 32 throughout

$$86 \le F \le 95$$

Therefore, the temperature changes from 86°F to 95°F.

EXERCISE 6.5

1. One-fourth of a number added to one-fifth of the same number is less than or equal to 18. Find the range of values of the number.

2. A petty trader wants to buy pineapples at ¢25 each and apples at ¢20 each. She decides to buy twice as many apples as pineapples. Her total cost was not less than L$ 6.50 and not more than L$ 7.80.

Taking x to be the number of pineapples:

(*a*) Write the given information as an inequality in x.

(*b*) Find the truth set of the inequality,

(*c*) Illustrate the solution set on the number line.

3. Henry scored 78 on his mid-semester examination in Mathematics. If he is to get a grade A, the average of his mid-semester and final examination must be between 80 and 88 inclusive. In what range must his final examination score lie to get a grade A?

REVIEW EXERCISE

1. Solve for x: $11 - 5x = 26$.

2. Solve for x: $\frac{x}{3} + 2 = -2$.

3. Solve for x:

(*a*) $4 = 3 - 2x$ (*b*) $13 = -1 - 7x$

4. Solve for x:

(*a*) $4 = \frac{2+x}{3}$ (*b*) $-1 + \frac{x}{3} = 7$

5. Solve the following equations and check your solution:

(*a*) $5x - 3 = 3x + 5$ (*b*) $3(x - 1) = x - 11$

(*c*) $3(2x - 1) = 5 - (3x - 2)$ (*d*) $2(x - 1) + 3 = x - 3(x + 1)$

6. Solve the following equations and check your solution:

(*a*) $\frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$ (*b*) $\frac{x-1}{3} - \frac{x-2}{4} = 1$

7. Shelia is four times as old as Albert. In ten years time, Shelia will be twice as old as Albert. Find their ages.

8. A boy has to cover 4 km to catch a bus. He walks part of the distance at 3 km/h and runs the rest at 5 km/h. If he takes 1 hour to complete the distance, for how many kilometres does he walk?

9. Solve the following inequalities and represent the solution on the number line:

(*a*) $8x + 3 > 3(2x + 1) + x + 5$ (*b*) $3(x - 2) \geq 2x - 3$

10. Solve the following inequalities and represent the solution on the number line:

(*a*) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$ (*b*) $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

11. 4 is added to three times a certain number, the result is not more than when twice that number added to 10. Find the possible range of values of the number.

12. The result of doubling a number is at most 20. What is the number?

MULTIPLE CHOICE QUESTIONS (MCQs)

1. The solution of the linear equation $-2(x + 3) = 8$ is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{9}{2}$ (d) None of these

2. The linear inequality $3x - 4 > 17$ has a solution given by

(a) $x > 7$ (b) $x \geq 7$ (c) $x < 7$ (d) $x \leq 7$

3. The value of x which satisfies the linear inequality $4x + 3 < 6x + 7$ is

(a) $x < -2$ (b) $x > -2$ (c) $x < 2$ (d) $x > 2$

4. The solution set of the linear inequality $5x - 3 < 7$ where x is an integer is

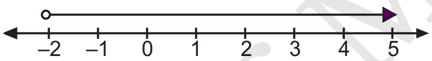
(a) {–4, –3, –2, –1, 0, 1} (b) {…, –4, –3, –2, –1, 0, 1}

(c) both (a) and (b) (d) None of these

5. The number line representing the solution set for the linear inequality $3x + 8 > 2$; x is a real number is

(a) 

(b) 

(c) 

(d) None of these

6. Albert says that he has 7 marbles more than five times the marbles Ella has. Albert has 37 marbles. How many marbles does Ella has?

(a) 7 (b) 8 (c) 6 (d) None of these

7. Islah's father is 49 years old. He is 4 years older than three times Islah age. The age of Islah's is

(a) 15 years (b) 19 years (c) 11 years (d) None of these

8. A teacher tells his class that the highest marks obtained by a pupil in his class is twice the lowest marks plus 7. The highest score is 87. The lowest score in the class is

(a) 80 (b) 47 (c) 87 (d) 40

9. Daniel subtracts thrice the number of notebooks he has from 50, he finds the result to be 8. The number of notebooks Daniel has

(a) 24 (b) 42 (c) 14 (d) None of these

10. The solution set of the linear inequality $5x + 8 < 23$; x is a whole number is

(a) {1, 2} (b) {0, 1} (c) {0, 2} (d) {0, 1, 2}

RECAP AT A GLANCE

- Two equations that have the same solution are called *equivalent* equations.
- Perform operations on both sides of each equation in order to *isolate the unknown.*
- *Solving an equation* is the process of finding the solutions of the equation.
- The set of all solutions of an equation is called the *solution set* of the equation.
- Two algebraic expressions related by the symbol < (is less than) or > (is greater than) or ≤ (is less than or equal to) or ≥ (is greater than or equal to) form an *inequality.*
- Problems stated in words are called *word problems.*
- Equations having the same solution set are called *equivalent equations.*
- If the same quantity is added to or subtracted from both sides of an inequality, then the sign of the inequality is not affected.
- If both sides of an inequality are multiplied or divided by a *positive number,* then the sign of inequality is not affected.
- If both sides of an inequality are multiplied or divided by a *negative number,* then the direction of inequality is reversed.
- A term may be transposed from one side of the inequality to the other side by changing its sign.
- Solutions of inequalities can be represented on a number line

❑ ❑ ❑